Engineering Notes

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Effect of Mesh Size on the Accuracy of Finite-Water Added Mass

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Nomenclature

B = beam of a cylinder
C_H = heave added mass coefficient
C_S = sway added mass coefficient
g = gravity constant
h = water depth
H = half beam draft ratio
T = draft of a cylinder
β = sectional area coefficient
2ρ = mesh size
ν = wave number ω^2/g
ω = circular wave frequency

THE accuracy of numerical calculations of the added mass of a heaving cylinder in water of finite depth is a topic of current interest in applied hydrodynamics. Various methods have recently been developed for evaluating the depth-influenced added mass of a cylinder: the multi-pole expansion method,[1-3] the variational method,[4,5] and the close-fit method.[6]

In the section "Numerical Calculation and Discussion" in Ref. 2, the author notes that he carried out the numerical integration of the singular functions $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$

$$G_{2s+1}(\nu h) = \oint_0^\infty (e^{-u} u^{2s+1} / (\nu h - u)(\nu h \cosh u - u \sinh u))\, du$$

$$F_{2s+1}(\nu h) = \oint_0^\infty (u^{2s+1}(u+\nu h) e^{-u} / \nu h \cosh u - u \sinh u) \tag{1}$$

according to the formula given by Yu and Ursell,[1] by taking the mesh size $(2\rho) = 0.5$. The νh in the equations is a nondimensional frequency parameter, $\omega^2 h/g$.

From the foregoing statement it is obvious that the effect of mesh size on the evaluation of added mass was not investigated in that work.[2] Attention has been paid by the author to the effect of mesh size during the investigation of relevant problems such as the evaluation of blockage factor of Lewis form cylinders and in a discussion in response to a private communication by Bai.[7] This Note, regarded as a supplement to Ref. 2, describes the effect of the mesh size for the singular integrals G and F on the accuracy of the added mass of the Lewis form cylinder.

The multipoles which represent the velocity potentials are (as shown in Ref. 2) expressed in infinite power series of the coordinates (r,β) of a point in the water with the coefficients $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$. These coefficients are dependent merely on the water depth h and the circular frequency of oscillation $\omega(\nu = \omega^2/g)$. This is a significant fact which has to be noted because the major part of the computer time is taken up with the evaluation of the singular functions. In applying the two-dimensional method[2] to the stripwise calculation of the added mass of a ship, it is necessary that in the calculation of every sectional mass one must use the same values of the singular functions $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$ at the frequency parameter νh.

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To investigate the effect of mesh size on the accuracy of heave added mass, a semicircular cylinder with depth-draft ratio $h/T = 2.0$ was chosen. Since the mesh size (2ρ) was 0.5 in Ref. 2, it has been changed to $(2\rho) = 0.1$ and 0.05. The effects of these are shown in Figs. 1-3. Figure 1 shows the behavior of $G_{2s+1}(\nu h)|_{s=0}$ for the mesh size $2\rho = 0.5$, 0.1, and 0.05, as functions of the frequency νh or $\nu B/2$ with the assumption $B/2 = 1.0$. Figure 2 shows the behavior of the heave added mass coefficient C_H in the low frequency range for 3 varying mesh sizes. In addition, the results are compared with the results by the variational method given by Bai.[7] Figure 3 shows the same heave added mass coefficients for the entire

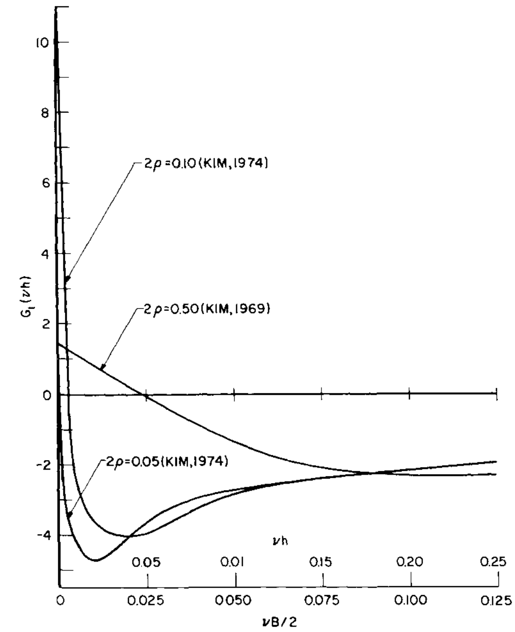

Fig. 1 Behavior of the singular function $G_1(\nu h)$ for varying (2ρ) at low frequencies.

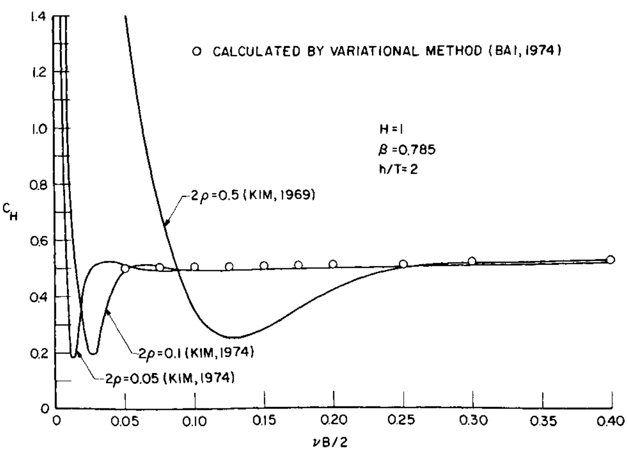

Fig. 2 Added mass coefficient for heave at low frequencies.

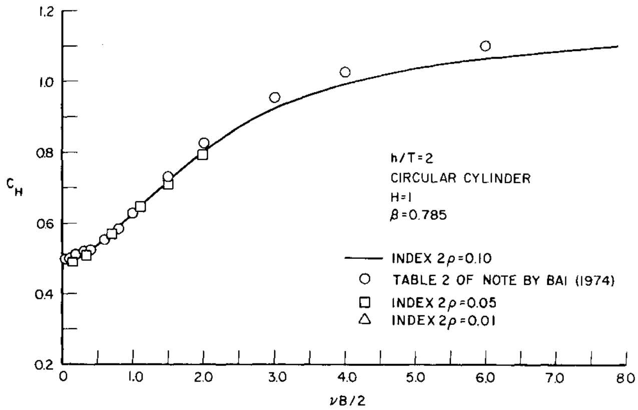

Fig. 3 Added mass coefficient for heave at entire frequency range.

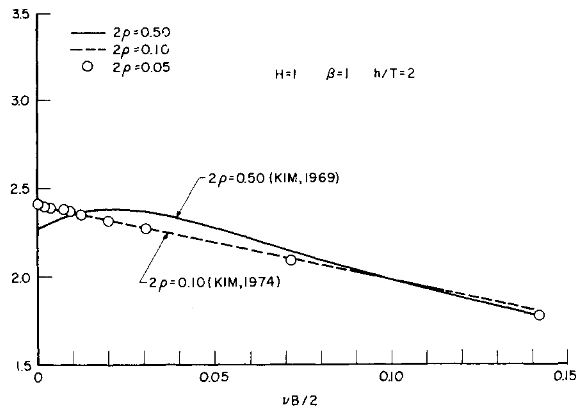

Fig. 4 Added mass coefficient for sway at low frequencies.

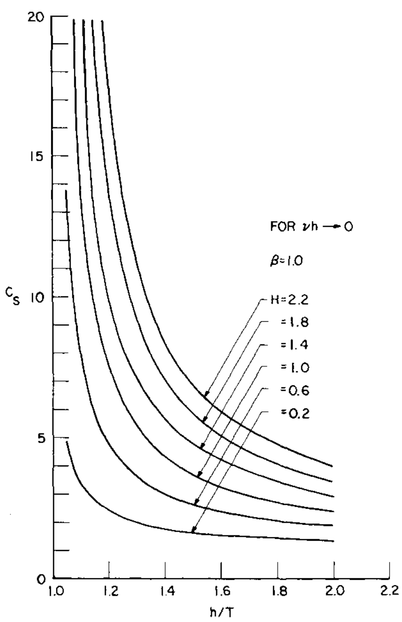

Fig. 5 Sway added mass coefficient of Lewis's form section.

frequency range. It also shows comparisons with Bai's results. It is seen from these figures that the accuracy of the heave added mass, referring to the results by Bai,[7] is dependent on the accuracy of the singular functions $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$ which are of course dependent on the mesh size (2ρ).

The effect of mesh size (2ρ) on the sway added mass in the low frequency range has also been studied. The results are shown in Fig. 4. It is seen from the figure that the effect of the mesh size on sway added mass is not as remarkably large as on the heave added mass. The sway added mass in the low frequency range for $\nu h \to 0$ is dependent on the $G_{2s+1} \times (\nu h)|_{s=1}$, which has been observed as less sensitive than the $G_{2s+1}(\nu h)|_{s=0}$ to the variation of mesh size although its behavior is not shown in Fig. 1.

Before the investigation of the accuracy of the added mass in the low frequency range, in fact two years ago, the author was faced with the need for evaluation of the so-called blockage factor which arises in the calculation of the steady forces on a ship moving laterally in shallow water.[8] Evaluation of the blockage factor requires the sway added mass of a cylinder for $\nu h \to 0$. Figure 5 shows the sway added mass coefficients C_S of a series of Lewis form sections, having $\beta = 1.0$ and $H = 2.0$-0.2, vs the depth-draft ratio h/T. The geometry of these sections is nearly rectangular. The coefficients C_S were in close agreement with the corresponding results for pure rectangular sections by Flagg and Newman.[9] Keil[3] also evaluated C_S for the same Lewis form cylinders.

Figure 1 shows that $G_1(\nu h)$ [Eq. (1)] for $\nu h \to 0$ approaches large positive values. It is not clear from the numerical calculation if the integral function is finite or infinite. Keil[3] shows that his analysis leads to negative infinity.

The heave added mass in finite water for vanishingly small frequency[2] is

$$C_{H}\underset{\substack{\nu h\to 0\\ h=h}}{\approx} C_{H}\underset{\substack{\nu h\to 0\\ h\to\infty}}{} + \frac{8}{\pi^2} G_{2s+1}(\nu h)\Big|_{\substack{s=0\\ \nu h\to 0}} \tag{2}$$

where

$$C_{H}\underset{\substack{\nu h\to 0\\ h\to\infty}}{\approx} \frac{8}{\pi^2}\left[-\ell n\left[\frac{\nu B}{2}\right] - 0.577\right] \tag{3}$$

and

$$G_1(\nu h)\underset{\nu h\to 0}{=} \ell n(\nu h) = 0.423 \qquad \text{(Ref. 3)}$$

Hence it is seen that the heave added mass coefficient C_H for $\nu h \to 0$ yields a finite value.

It is concluded from this study that: 1) The mesh size (2ρ) for evaluating the singular functions $G_{2s+1}(\nu h)$ and $F_{2s+1}(\nu h)$ must be, for practical purposes, less than 0.05. 2) The heave mass coefficients C_H for $\nu h \to 0$ is bounded.

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